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HEAT AND MASS TRANSFER TO A TRANSLATING DROP IN AN ELECTRIC FIELD

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Abstract—The transient and quasi-steady heat and mass exchange between a translating drop and its surroundings in a uniform electric field is investigated for the case of low Reynolds and high Peclet numbers. The energy or diffusion equation for the case of thin thermal or diffusion boundary layers is solved by similarity transformation and numerical schemes, and results for the rate of heat or mass transfer are obtained. A parameter W, which characterizes the relative importance of electrical and gravitational effects, emerges. It is found that only when the absolute value of W is larger than unity is the rate of transport enhanced by the imposed electric field. The relationship between the flow patterns and the rate of transport is discussed.

NOMENCLATURE

C,	solute concentration;
c,	constant in equation (26);
D,	mass diffusivity;
E_0,	uniform electric field strength;
f,	function defined in equation (21);
g,	gravitational constant;
I,	integral defined in equation (38);
I_{ss},	integral defined in equation (40);
K,	distribution coefficient;
K',	$= (k_1/k_2)(\kappa_2/\kappa_1)^{1/2}$ for heat transfer and $(D_1/D_2)^{1/2}$ for mass transfer;
k,	thermal conductivity;
Nu,	Nusselt number;
Pe,	Peclet number; $2\|U\|R/\kappa_1$ for heat transfer and $2\|U\|R/D_1$ for mass transfer;
Pe',	$Pe/4(1+X)$;
Q,	total rate of heat transfer;
q,	local heat flux;
R,	drop radius;
r,	radial coordinate;
T,	temperature;
t,	time;
U,	terminal velocity of the drop;
u_r,	radial velocity;
u_θ,	tangential velocity;
V,	maximum speed generated by electric field;
W,	dimensionless parameter defined in equation (4);
X,	viscosity ratio; μ_2/μ_1;
y,	distance from interface;
Z,	dimensionless temperature or concentration; $(T - T_\infty/T_0 - T_\infty)$ $(C_1 - C_\infty/C_0 - KC_\infty)$ or $(C_2 - KC_\infty/C_0 - KC_\infty)$.

Greek symbols

ε,	permittivity;
ζ,	similarity variable defined in equation (22);
η,	similarity variable defined in equation (21);
θ,	polar angle as in Fig. 1(a);
κ,	thermal diffusivity;
μ,	viscosity;
ξ,	dimensionless form of y: $\xi_1 = y/R$, $\xi_2 = (y/R)(\kappa_1/\kappa_2)^{1/2}$ or $(y/R)(D_1/D_2)^{1/2}$;
ρ,	density;
σ,	electrical resistivity;
τ,	dimensionless time; $\kappa_1 t/R^2$ or $D_1 t/R^2$;
τ_{ij},	stresses;
ψ,	stream function;
$\hat{\psi}$,	dimensionless stream function.

Subscripts

1,	continuous phase;
2,	drop phase;
E,	electric field;
ss,	steady state.

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INTRODUCTION

THE CIRCULATION developed within a drop moving under the influence of gravity through a viscous medium, first described by Hadamard and Rybczynski and shown in Fig. 1(a), has been shown by later investigators [1–3] to increase significantly the rate of heat or mass exchange between the drop and its surroundings. Taylor [4], on the other hand, considered a stationary drop of a leaky dielectric fluid in another such fluid in the presence of a uniform electric field. The interaction of the electric field with the charge which accumulates at the interface produces a tangential stress distribution leading to a different type of circulation, shown in Fig. 1(b). Circulation could be either from the equator toward the poles (for

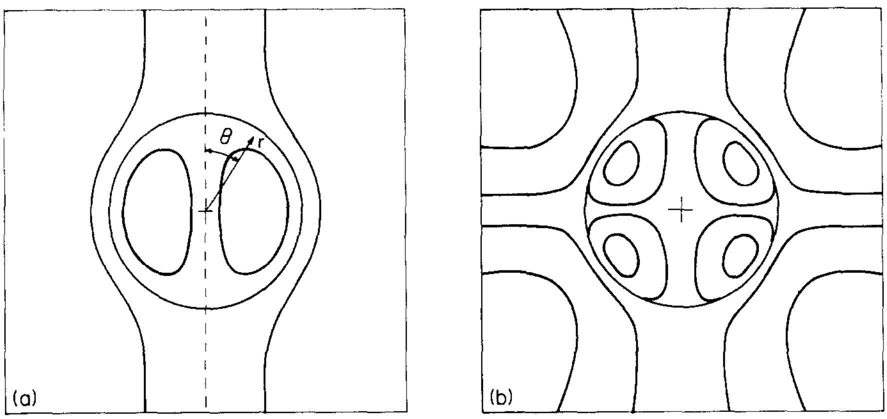

FIG. 1. Flow patterns and co-ordinate system. (a) Hadamard–Rybczynski flow pattern showing spherical coordinates used in the analysis, (b) Taylor flow pattern for a stationary drop in a uniform electric field.

$\varepsilon_1\sigma_1 > \varepsilon_2\sigma_2$) or from the poles toward the equator (for $\varepsilon_1\sigma_1 < \varepsilon_2\sigma_2$). Morrison [5] and Griffiths and Morrison [6] later computed the extent to which the Taylor circulation enhanced heat or mass exchange and speculated that the use of electric fields might thus show promise of yielding more compact exchangers. Experimental results of Thornton [7, 8] and Bailes and Thornton [9] for electrified liquid extraction units appear to corroborate such speculation. An analysis fully relevant to Thornton's experiments, or indeed to any electrified liquid extraction or direct contact heat exchange unit in practice, however, must describe the hybrid circulation resulting from both the electric field *and* gravitational settling. The present work seeks to provide such a description, to determine the conditions under which one type of circulation may dominate over the other, and finally to reassess the promise of heat and mass exchange enhancement through the use of electric fields. Unsteady and quasi-steady transfer rates are obtained for the case of low Reynolds number (creeping flow) and high Peclet number (thin boundary layer) using a similarity transform method developed by Chao [2] and employed by Morrison [5]. The fluids are assumed Newtonian with constant properties, and the drop is assumed to remain spherical under all conditions.

ANALYSIS

Flow patterns

Steady creeping flow in both the outside phase (phase 1) and inside phase (phase 2) are described by

$$\left[\frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)\right]^2 \psi_i = 0, \quad i = 1, 2 \quad (1)$$

with boundary conditions

$$\psi_1 \to \tfrac{1}{2}|U|r^2\sin^2\theta, \quad \text{as } r \to \infty$$

$$\psi_1 = \psi_2 = 0, \quad \text{at } r = R$$

$$u_{\theta_1} = u_{\theta_2}, \quad \text{at } r = R$$

$$U_{r_2}, u_{\theta_2} \neq \infty, \quad \text{at } r = 0$$

$$\tau_{r\theta_E} + \tau_{r\theta_1} - \tau_{r\theta_2} = 0, \quad \text{at } r = R$$

$$\tau_{rr_E} + \tau_{rr_1} - \tau_{rr_2} = \text{constant}, \quad \text{at } r = R.$$

The last two conditions are for continuity of tangential and normal stresses respectively, and include the electrical stresses $\tau_{r\theta_E}$ and τ_{rr_E}. Integration of equation (1) leads to

$$\psi_1 = |U|R^2\left(\frac{1}{2}\frac{r^2}{R^2} - \frac{3}{4}\frac{\frac{2}{3}+X}{1+X}\frac{r}{R} + \frac{1}{4}\frac{X}{1+X}\frac{R}{r}\right)\sin^2\theta + VR^2\left(\frac{R^2}{r^2} - 1\right)\sin^2\theta\cos\theta \quad (2)$$

and

$$\psi_2 = |U|R^2\frac{1}{4(1+X)}\left(\frac{r^4}{R^4} - \frac{r^2}{R^2}\right)\sin^2\theta + VR^2\left(\frac{r^3}{R^3} - \frac{r^5}{R^5}\right)\sin^2\theta\cos\theta \quad (3)$$

where

$$V = \frac{9E_0^2 R\varepsilon_2\left(1 - \dfrac{\varepsilon_1\sigma_1}{\varepsilon_2\sigma_2}\right)}{10\left(2 + \dfrac{\sigma_1}{\sigma_2}\right)^2\mu_1(1+X)},$$

$$U = \frac{2gR^2(\rho_2 - \rho_1)(1+X)}{3\mu_1(2+3X)}.$$

V is the maximum surface velocity generated by the electric field and in the absence of circulation due to gravity, occurs at $\theta = \pi/4$. U is the terminal velocity of the drop falling in a medium of infinite extent in the absence of an electric field. Because the flow induced by the electric field contributes no net force to the drop, the terminal velocity of the drop remains the same as that of a drop falling by gravity alone. The absolute sign for U is used to accommodate either a rising or falling drop. Defining

$$W = \frac{4V(1+X)}{|U|}$$

$$= \frac{27\varepsilon_2\left(1-\frac{\varepsilon_1\sigma_1}{\varepsilon_2\sigma_2}\right)(2+3X)}{5|\rho_2-\rho_1|g\left(2+\frac{\sigma_1}{\sigma_2}\right)^2(1+X)}\cdot\frac{E_0^2}{R} \quad (4)$$

and non-dimensionalizing the stream functions by dividing them by $|U|R^2/4(1+X)$, we obtain

$$\hat{\psi}_1 = \left[(2+2X)\frac{r^2}{R^2} - (2+3X)\frac{r}{R} + X\frac{R}{r}\right]\sin^2\theta + W\left(\frac{R^2}{r^2}-1\right)\sin^2\theta\cos\theta \quad (5)$$

and

$$\hat{\psi}_2 = \left(\frac{r^4}{R^4}-\frac{r^2}{R^2}\right)\sin^2\theta + W\left(\frac{r^3}{R^3}-\frac{r^5}{R^5}\right)\sin^2\theta\cos\theta. \quad (6)$$

The first terms on the right-hand side of equations (5) and (6) are the expressions for the Hadamard–Rybczynski flow fields. The last terms of equations (5) and (6) represent the internal and external circulation caused by the electric effect. W is a dimensionless parameter that characterizes the relative importance of the electric field and the translation of the drop due to gravity. Figure 2 illustrates the dependence of the flow pattern on W. A positive W means the electrically-induced surface flow is from pole to equator, while for negative W it is reversed. As $|W|$ increases from 0, corresponding to no electric field, the inception of the double torus flow occurs at $|W| = 1.0$, and for values of $|W|$ larger than 10 the inside flows differ little from the case of no translational motion. Extraction of the velocity components from the stream functions for small values of $y = r - R$ and neglecting terms of higher order in y/R leads to

$$u_{r_1} = u_{r_2} = 2V(\sin^2\theta - 2\cos^2\theta)\frac{y}{R} + \frac{|U|}{1+X}\cos\theta\frac{y}{R} \quad (7)$$

and

$$u_{\theta_1} = u_{\theta_2} = 2V\sin\theta\cos\theta - \frac{|U|}{2(1+X)}\sin\theta. \quad (8)$$

We may note that $\theta = \cos^{-1}(1/W)$ locates the stagnation line on the interface.

Boundary layer equations

We now consider a drop moving in both gravitational and electric fields with a fully-developed flow pattern as described above. Initially both phases are at a uniform and constant temperature T_0. We seek to describe the transient response of the system if at $t = 0$ the temperature of the continuous phase is changed to T_∞ and remains constant thereafter.

An order-of-magnitude analysis leads to the following form of the energy equation for the assumption of a thin thermal boundary layer on each side of the drop interface

$$\frac{\partial T_i}{\partial t} + u_{r_i}\frac{\partial T_i}{\partial y} + \frac{u_{\theta i}}{R}\frac{\partial T_i}{\partial \theta} = \kappa_i\frac{\partial^2 T_i}{\partial y^2} \quad i = 1, 2. \quad (9)$$

This approximation greatly simplifies the energy equation and is valid as long as the Peclet number is large, i.e. $|y|/R \sim 1/\sqrt{Pe}$, except near the stagnation points, at $\theta = 0, \pi$, and $\cos^{-1}(1/W)$. It thus suffers the same limitations as the earlier analyses of Chao [2] and Morrison [5].

Combining equation (9) with equations (7) and (8), we obtain

$$\frac{\partial T_i}{\partial t} + [W(\sin^2\theta - 2\cos^2\theta) + 2\cos\theta]\frac{|U|}{2(1+X)}\frac{y}{R}\frac{\partial T_i}{\partial y} + (W\sin\theta\cos\theta - \sin\theta)\frac{|U|}{2(1+X)}\frac{1}{R}\frac{\partial T_i}{\partial\theta} = \kappa_i\frac{\partial^2 T_i}{\partial y^2}, \quad i = 1, 2. \quad (10)$$

The initial and boundary conditions are

$$T_1(y, \theta, 0) = T_\infty, \qquad T_2(y, \theta, 0) = T_0 \quad (11)$$

$$T_1(\infty, \theta, t) = T_\infty, \qquad T_2(-\infty, \theta, t) = T_0 \quad (12)$$

$$T_1(0, \theta, t) = T_2(0, \theta, t) \quad (13)$$

$$k_1\frac{\partial T_1}{\partial y}(0, \theta, t) = k_2\frac{\partial T_2}{\partial y}(0, \theta, t) \quad (14)$$

together with symmetry conditions at $\theta = 0$ and π. It is the second of equations (12) which appears to restrict the analysis to relatively short times. The $\pm\infty$ in these equations refers to conditions well removed from the interface, compared with the boundary layer thickness, but still to values of $|y| \ll R$.

For the analogous problem of mass transfer, one need only replace T by C, etc. The continuity of temperature at the interface, equation (13), is replaced by

$$KC_1(0, \theta, t) = C_2(0, \theta, t). \quad (15)$$

Non-dimensionalization of the equations and boundary conditions yields

$$\frac{\partial Z}{\partial \tau} + Pe'[W(\sin^2\theta - 2\cos^2\theta) + 2\cos\theta]\xi_i\frac{\partial Z}{\partial \xi_i} + Pe'(W\sin\theta\cos\theta - \sin\theta)\frac{\partial Z}{\partial\theta} = \frac{\partial^2 Z}{\partial \xi_i^2}, \quad i = 1, 2 \quad (16)$$

with

$$Z_1(\xi_1, \theta, 0) = 0, \qquad Z_2(\xi_2, \theta, 0) = 1 \quad (17)$$

$$Z_1(\infty, \theta, \tau) = 0, \qquad Z_2(-\infty, \theta, \tau) = 1 \quad (18)$$

$$KZ_1(0, \theta, \tau) = Z_2(0, \theta, \tau) \quad (K = 1 \text{ for heat transfer}) \quad (19)$$

and

$$K' \frac{\partial Z_1}{\partial \xi_1}(0, \theta, \tau) = \frac{\partial Z_2}{\partial \xi_2}(0, \theta, \tau). \quad (20)$$

Since equation (16) is parabolic and there is no reference time or length scale, we may introduce a similarity transformation following Chao [2]. We put

$$Z_i = Z_i(\eta_i, \zeta), \quad i = 1, 2$$

where

$$\eta_i = \xi_i f(\theta) \quad (21)$$

and

$$\zeta = \zeta(\theta, \tau). \quad (22)$$

Equation (16) is thus transformed into

$$\left[\frac{\partial \zeta}{\partial \tau} + Pe'(W \sin\theta \cos\theta - \sin\theta)\frac{\partial \zeta}{\partial \theta}\right]\frac{\partial Z}{\partial \zeta} + Pe'\left[W(\sin^2\theta - 2\cos^2\theta) + 2\cos\theta + \frac{1}{f}\frac{df}{d\theta}(W\sin\theta\cos\theta - \sin\theta)\right]\eta_i \frac{\partial Z_i}{\partial \eta_i} = f^2 \frac{\partial^2 Z_i}{\partial \eta_i^2}, \quad i = 1, 2. \quad (23)$$

The functions f and ζ are to be chosen such that

$$W(\sin^2\theta - 2\cos^2\theta) + 2\cos\theta = -\frac{1}{f}\frac{df}{d\theta}\sin\theta\,(W\cos\theta - 1) \quad (24)$$

and

$$\frac{\partial \zeta}{\partial \tau} + Pe' \sin\theta\,(W\cos\theta - 1)\frac{\partial \zeta}{\partial \theta} = f^2. \quad (25)$$

Equation (24) gives

$$f = c\sin^2\theta\,(W\cos\theta - 1) \quad (26)$$

in which c can be chosen arbitrarily. For simplicity we may choose $c^2 = Pe'$ and substitute equation (26) into (25). The resulting first-order linear partial differential equation is

$$\frac{\partial \zeta}{\partial \tau} + Pe' \sin\theta\,(W\cos\theta - 1)\frac{\partial \zeta}{\partial \theta} = Pe' \sin^4\theta\,(W\cos\theta - 1)^2 \quad (27)$$

with

$$\zeta(\theta, 0) = 0. \quad (28)$$

This equation is much easier to solve than the untransformed equations, but closed form solutions can be obtained only for certain values of W. In order not to lose generality, we solve it using a finite difference scheme. The spatial derivative is approximated by a second-order-accurate central difference expression, while the time derivative is either approximated by a first-order finite difference expression or by a Runge–Kutta scheme. One of the grid points is located at the singularity for the case $|W| > 1$.

Now the problem is reduced to solving for Z_i as a function of η_i and ζ. Equation (23) becomes

$$\frac{\partial Z_i}{\partial \zeta} = \frac{\partial^2 Z_i}{\partial \eta_i^2}, \quad i = 1, 2 \quad (29)$$

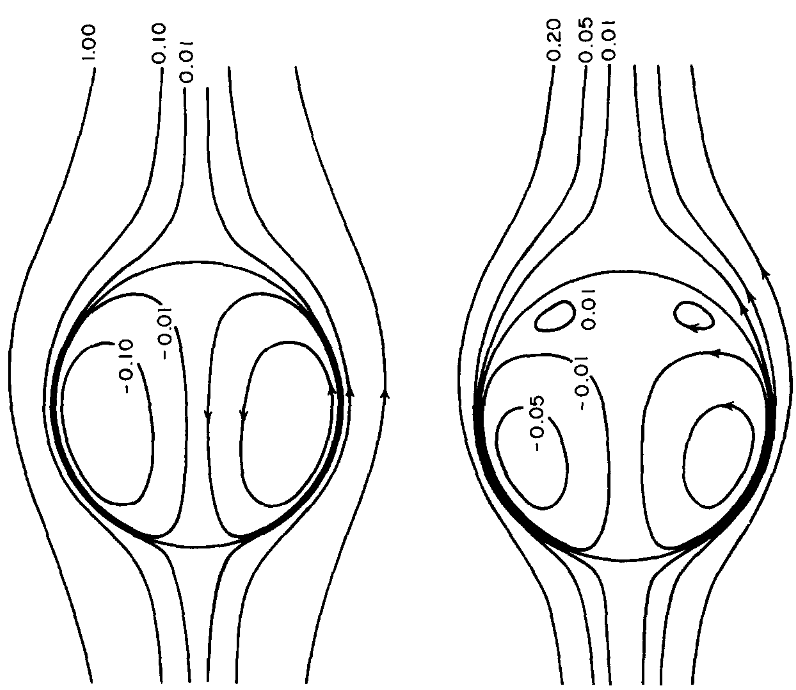

FIG. 2(a). $W = 1.0$. FIG. 2(b). $W = 2.0$.

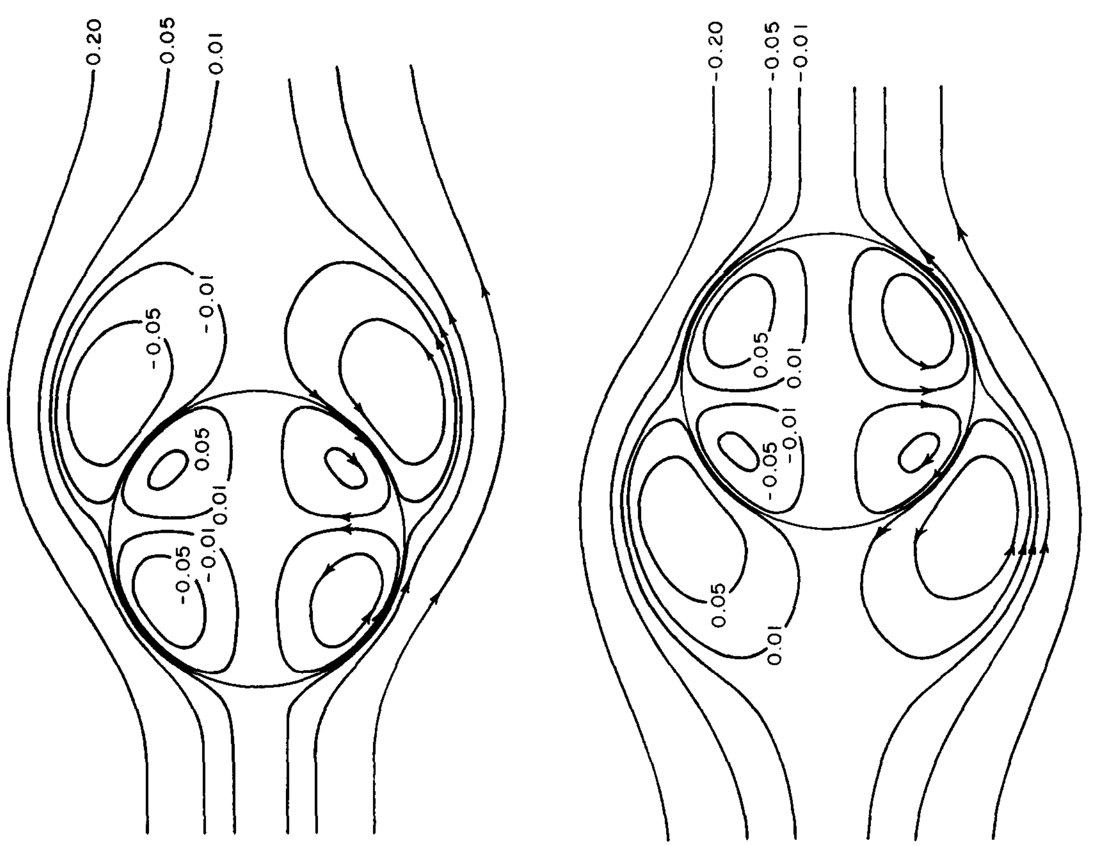

FIG. 2(c) $W = 10.0$. FIG. 2(d). $W = 10.00$ and $X = 1$.

FIG. 2. Computed flow patterns in the drop phase and the continuous phase for several values of W. Numbers show values of the dimensionless stream function, ψ_i/VR^2.

with

$$Z_1(\eta_1, 0) = 0, \qquad Z_2(\eta_2, 0) = 1 \tag{30}$$

$$Z_1(\infty, \zeta) = 0, \qquad Z_2(-\infty, \zeta) = 1 \tag{31}$$

and

$$KZ_1(0, \zeta) = Z_2(0, \zeta),$$

$$K'\frac{\partial Z_1}{\partial \eta_1}(0, \zeta) = \frac{\partial Z_2}{\partial \eta_2}(0, \zeta) \tag{32}$$

This equation is solved in Carslaw and Jaeger [10], viz.

$$Z_1 = \frac{1}{K+K'}\ \text{erfc}\ \frac{|\eta_1|}{2\sqrt{|\zeta|}} \tag{33}$$

$$Z_2 = 1 - \frac{K'}{K+K'}\ \text{erfc}\ \frac{|\eta_2|}{2\sqrt{|\zeta|}} \tag{34}$$

with

$$\eta_i = (Pe')^{1/2} \sin^2\theta\,(W\cos\theta - 1)\,\xi_i$$

Some examples of dimensionless temperature or concentration profiles are shown in Fig. 3.

For larger times, ζ approaches a 'quasi-steady' value. Thus

$$\zeta_{ss} = \tfrac{1}{4}W\sin^4\theta + \cos\theta - \tfrac{1}{3}\cos^3\theta + \tfrac{2}{3} \text{ for positive } W. \tag{35}$$

Similar expressions can be obtained if from the beginning we drop the temporal term in the energy equation. Equation (35) shows clearly the role of the parameter W as a weighting factor for the relative importance of the effects of the electric field and gravity. The result for negative W* is obtained in the same way.

Heat transfer

Since the analysis for mass transfer is essentially the same as that for heat transfer, we concentrate here on the latter. The results hold for the case of mass transfer if suitable substitutions are made.

* For negative W equation (35) becomes

$$\zeta_{ss} = \tfrac{1}{4}W\sin^4\theta + \cos\theta - \tfrac{1}{3}\cos^3\theta - \tfrac{2}{3}. \tag{35a}$$

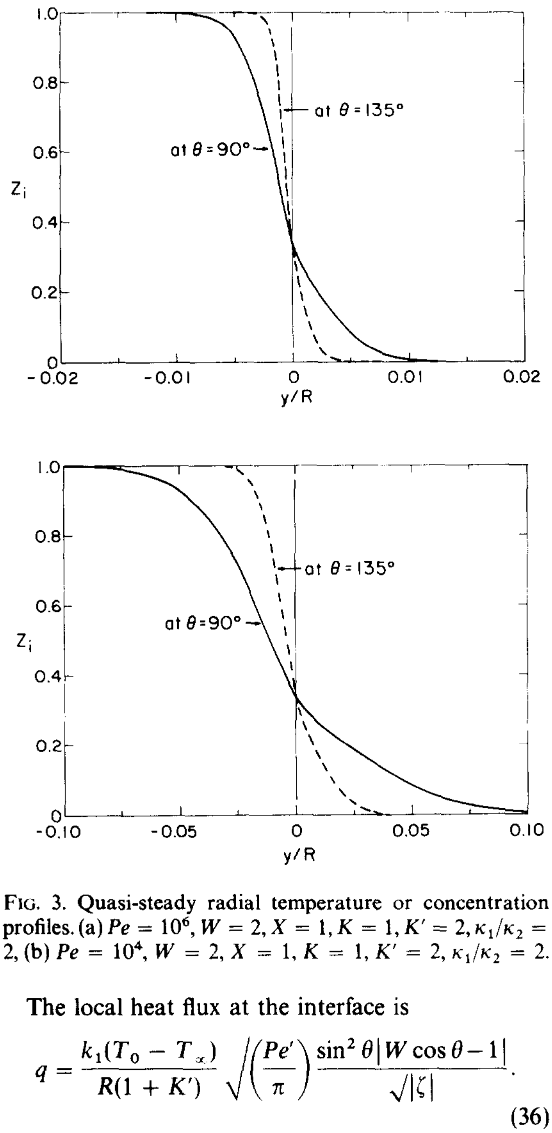

FIG. 3. Quasi-steady radial temperature or concentration profiles. (a) $Pe = 10^6, W = 2, X = 1, K = 1, K' = 2, \kappa_1/\kappa_2 = 2$, (b) $Pe = 10^4, W = 2, X = 1, K = 1, K' = 2, \kappa_1/\kappa_2 = 2$.

The local heat flux at the interface is

$$q = \frac{k_1(T_0 - T_\infty)}{R(1 + K')} \sqrt{\left(\frac{Pe'}{\pi}\right)} \frac{\sin^2\theta |W\cos\theta - 1|}{\sqrt{|\zeta|}}. \tag{36}$$

The local Nusselt number is correspondingly

$$Nu_\theta = \frac{2}{1+K'} \sqrt{\left(\frac{Pe'}{\pi}\right)} \frac{\sin^2\theta |W\cos\theta - 1|}{\sqrt{|\zeta|}}. \tag{37}$$

The total rate of heat transfer between the two phases is

$$Q = 2\pi R^2 \int_0^\pi q \sin\theta \, d\theta$$

with which the overall Nusselt number over the interface is calculated to be

$$Nu = \left[\frac{4}{\sqrt{3}} \frac{(Pe'/\pi)^{1/2}}{1 + K'}\right] I \tag{38}$$

where

$$I = \frac{\sqrt{3}}{4} \int_0^\pi \frac{\sin^3\theta |W\cos\theta - 1| d\theta}{\sqrt{|\zeta|}}$$

The grouping in the bracket of equation (38) is identical to the steady state average Nusselt number of Chao [2]. Thus

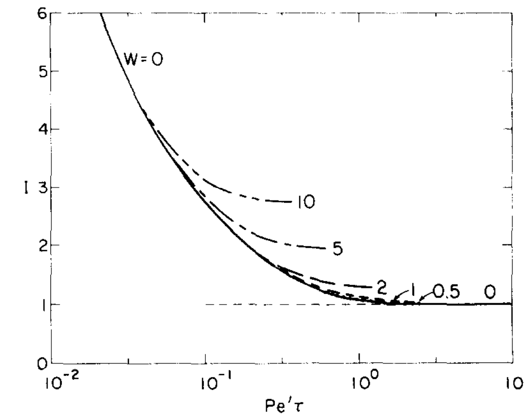

FIG. 4. Ratio of transient Nusselt number at different values of W to the corresponding quasi-steady Nusselt number for a drop translating in a gravitational field only as a function of modified dimensionless time.

$$\frac{Nu}{Nu_{ss, W=0}} = I. \tag{39}$$

The integral I is evaluated by Gaussian quadrature. The transient behavior of Nu for several values of W is shown in Fig. 4. The line for $W = 0$ corresponds to a drop translating in a gravitational field only. For all values of W the curves approach the quasi-steady-state values as τ becomes large. In this analysis the transient behavior is independent of Peclet number, as long as the latter is large enough for the thin boundary layer assumption to hold. All the curves coincide at short times, a fact indicating the predominance of conduction under these conditions [3]. The effect of convective heat transfer becomes perceptible after a certain contact time which depends on W. The Nusselt number behavior for all values of $|W| \leq 1$ is essentially the same. This is directly related to the observation that the flow pattern remains essentially that of Hadamard–Rybczynski as long as $|W|$ is ≤ 1.

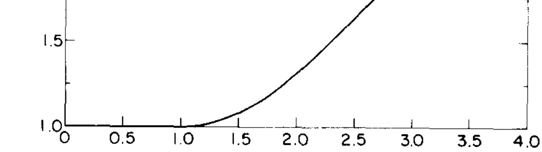

FIG. 5. Ratio of quasi-steady Nusselt number for a drop in both electric and gravitational fields to the corresponding value for a drop translating in a gravitational field only as a function of $\sqrt{|W|}$, which in turn is proportional to the electric field strength.

We may also calculate the overall quasi-steady Nusselt number. We have

$$Nu_{ss} = \frac{4}{\sqrt{3}} \frac{(Pe'/\pi)^{1/2}}{1+K'} I_{ss}$$

where

$$I_{ss} = \frac{\sqrt{3}}{4} \int_0^{\pi} \frac{\sin^3\theta |W\cos\theta - 1| d\theta}{\sqrt{|\zeta_{ss}|}}$$

or

$$\frac{Nu_{ss}}{Nu_{ss,W=0}} = I_{ss} \tag{40}$$

I_{ss} is evaluated by either Gaussian quadrature or a Runge–Kutta procedure and is plotted in Fig. 5. The overall steady-state Nusselt number is independent of the direction of flow, i.e. I_{ss} is identical for either a positive or negative W of given magnitude. Since W is proportional to E_0^2, $|W|^{1/2}$ is proportional to the imposed electric field strength. It is seen that as $|W|$ becomes large, I_{ss} is linearly proportional to $|W|^{1/2}$ or E_0, which is the limiting case of Morrison [5]. As W approaches zero, I_{ss} remains at unity and coincides with the result of Chao [2] for the case of drop translation in the absence of an electric field. It is noteworthy that no significant enhancement of transfer occurs until $|W|$ is well above unity.

In Fig. 6, I_{ss} is evaluated with the upper limit replaced by an angle ranging from 0 to π to reveal the contribution of different regions of the surface to the total heat transfer rate. The relationship of these curves to the internal and external circulatory patterns is evident. For the cases of $|W| > 1$, the double torus circulation pattern exists, and the integration from 0 to $\theta = \cos^{-1}(1/W)$ yields exactly half of $(I_{ss} - 1)$. This means that when a strong electric field enhances convective heat transfer, each torus contributes equally to the enhancement, regardless of their relative sizes.

DISCUSSION AND CONCLUSIONS

Experimental results of Thornton [7, 8] and Bailes and Thornton [9] for mass transfer in electrified liquid extraction units have been cited as evidence for the significant transfer rate enhancements which could be achieved by using electric fields. From such results it might be speculated that a Taylor type of induced internal circulation in the drops is responsible for the observed enhancement. The results of the present analysis now permits assessment of such speculations. Although not all of the physical parameters of the experimental systems were given explicitly, we were able to calculate values of W for most of them. In no case was W found to be greater than 1.0, and in most of them it was very near to zero. We may thus conclude that electrically induced internal circulation played no role in the mass transfer enhancement observed. One possible explanation may be related to the fact that in the experiments, the drops bore a net electrical charge and were thus drawn through the medium at higher velocity. The average drop size might also have been reduced, increasing interfacial area. Finally, droplet shape oscillation might have been a factor. Sufficient data are not available to fully check these possibilities. More recent studies [11, 12] have employed different electric field geometries and fields changing with time, but in neither case does it appear that observed enhancements of mass transfer can be attributed to changes in internal circulation.

It is possible to select a system with a high value of W. For instance, benzonitrile and water under suitable conditions constitute a system that yields a W as high as several hundred for a drop of 2 mm dia. under an electric field of a few hundred kV m^{-1}. Even under these conditions, however, it is not possible to ignore other effects. Field strengths higher than the above generally lead to drop bursting or in any event are impractical in view of energy consumption considerations. Taylor's original assumptions require that the drop phase have a small electrical relaxation time, defined as $\varepsilon\sigma$, such that the surface charge distribution is established instantaneously, and that the continuous phase have a large relaxation time in order to minimize charge leakage. On the other hand, we know from equation (4) that W will be large if $\sigma_1 \ll \sigma_2$ or ε_2 is large, both of which contradict these criteria. From a practical point of view, it is also required that σ_1 be large in order to minimize energy consumption. These considerations suggest that under usual practical conditions, it is unlikely that electric circulation can play a significant role in enhancing mass or heat transfer. Thus, although it has been shown by Morrison [5] and Griffiths and Morrison [6] that the electric circulation can greatly enhance the heat or mass transfer to a stationary drop in an electric field, this enhancement is not so likely in the more realistic situation in which the drop is also moving in a gravitational field.

It is important to note that we have considered only the case of an electric field parallel to translation. A

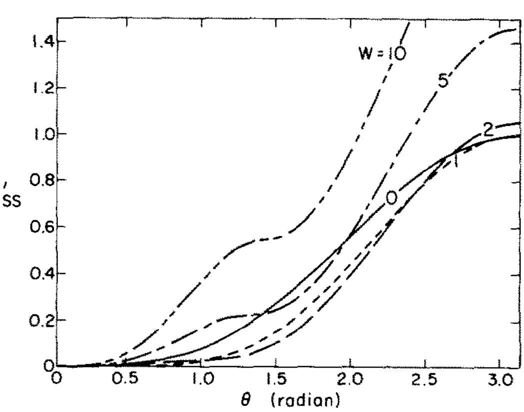

FIG. 6. Total heat transfer over successive regions of the drop surface for various values of W showing the effect of electrically-induced internal circulation

$$I'_{ss} = \frac{\sqrt{3}}{4} \int_0^{\alpha} \frac{\sin^3\theta |W\cos\theta - 1| d\theta}{\sqrt{|\zeta_{ss}|}}, \quad 0 \le \alpha \le \pi.$$

field perpendicular to translation, for example, may have markedly different effect. This case will be examined in a later note.

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TRANSFERT DE CHALEUR ET DE MASSE A UNE GOUTTE EN TRANSLATION DANS UN CHAMP ELECTRIQUE

Résumé—L'échange de chaleur et de masse transitoire et quasi-statique entre une goutte en translation et son environnement, dans un champ électrique uniforme, est étudié dans le cas de nombre de Reynolds faibles et de nombres élevés de Péclet. L'équation d'énergie ou de diffusion pour le cas de couches limites minces thermique ou massique est résolue par une transformation en similitude et des schémas numériques, et on obtient des résultats sur le transfert de chaleur et de masse. On dégage un paramètre W qui caractérise l'importance relative des effets électriques et gravitationnels. On trouve que seulement lorsque la valeur absolue de W est plus grande que l'unité, le champ électrique imposé augmente le transfert. On discute la relation entre la configuration de l'écoulement et le transfert.

WÄRME- UND STOFFÜBERTRAGUNG AN EINEN BEWEGTEN TROPFEN IM ELEKTRISCHEN FELD

Zussamenfassung—Der instationäre und quasi-stationäre Wärme- und Stoffaustausch zwischen einem bewegten Tropfen und seiner Umgebung in einem gleichförmigen elektrischen Feld wurde für den Fall kleiner Reynolds- und hoher Peclet-Zahlen untersucht. Die Energie- bzw. Diffusionsgleichung wurde für den Fall einer dünnen thermischen bzw. Diffusions-Grenzschicht durch Ähnlichkeitstransformation und numerische Methoden gelöst, und es wurden Ergebnisse für den Wärme- bzw. Stoffübergang erhalten. Ein Parameter W, der den relativen Einfluß von elektrischen und Schwerkrafteffekten charakterisiert, wurde ermittelt. Es wurde gefunden, daß die Transportrate nur dann durch das aufgeprägte elektrische Feld erhöht wird, wenn der absolute Wert von W größer als eins ist. Der Zusammenhang zwischen den Strömungszuständen und der Transportrate wird diskutiert.

ТЕПЛО- И МАССОПЕРЕНОС К КАПЛЕ, ПЕРЕМЕЩАЮЩЕЙСЯ В ЭЛЕКТРИЧЕСКОМ ПОЛЕ

Аннотация — Исследован нестационарный и квазистационарный тепло- и массообмен между перемещающейся каплей и окружающей средой в однородном электрическом поле при малых значениях числа Рейнольдса и больших числах Пекле. На основе теории подобия численно решено уравнение энергии и диффузионное уравнение для случая тонких соответственно теплового и диффузионного погранслоев и получены данные для теплового и диффузионного потоков. Введен параметр W, характеризующий соотношение электрических и гравитационных сил. Установлено, что внешнее электрическое поле интенсифицирует перенос только для $|W| > 1$. Рассмотрена связь структуры потока и скорости переноса.